## Substitute Section of Specification

### *I. PROBLEM FORMULATION*

Referring to **Figure 1**, a receiver **10** is configured to receive signal *r*(t) **20**, which is a linear combination of a set of signature signals $\{s_k(t), 1 \le k \le M\}$ and a noise component $n(t)$. The received signal *r*(t) **20** is processed by a bank of correlators **30**, which may, for example, be a matched filter or a decorrelator receiver. The received signal *r*(t) **20** is cross-correlated with $M$ signals $y_m$(t) **90** so that the vector output has components $a_m = \langle y_m(t), r(t) \rangle$ (inner product), where the signals $y_m$(t) **90** are to be determined. The vector output **a 40** may then be additionally shaped by a correlation shaper **50**. The vector output **x 60** of the correlation shaper could be passed to a detector **70** or similar device.

Described below are numerous embodiments of the present invention. Several of the embodiments presume that the bank of correlators **30** comprises a decorrelator receiver, and others presume the bank of correlators **30** comprises a matched filter receiver. One skilled in the art, however, will recognize that the bank of correlators **30** is not limited to being either a decorrelator receiver or a matched filter receiver. While these embodiments are physically distinct, many of their solutions are mathematically equivalent. For a discussion of this mathematical equivalence, see the co-pending patent application entitled "Correlation Shaping Matched Filter Receiver" filed February 20, 2001, and assigned to the assignee of the present application, and Y.C. Eldar, A.V. Oppenheim, and D. Egnor, "Orthogonal and Projected Orthogonal Matched Filter Detection," submitted to *IEEE Trans. on Signal Proc.* January 2001. Both of these works are hereby incorporated by reference in their entirety. In the notation that follows, the symbol **W** is used with reference to a transformation function.

Consider an $M$ user white Gaussian synchronous CDMA system. The received signal $r(t)$ **20** is given by

$$r(t) = \sum_{m=1}^{M} A_m b_m s_m(t) + n(t),$$

where $s_m(t)$ is the signature signal of the $m^{\text{th}}$ user, $A_m > 0$ is the received amplitude of the $m^{\text{th}}$ user's signal, $b_m$ is a symbol transmitted by the $m^{\text{th}}$ user, and $n(t)$ is a white noise signal with zero mean and covariance $\sigma^2$.

Based on the received signal $r(t)$ **20**, a receiver may be designed to demodulate the information transmitted by each user. We restrict our attention to linear receivers that do not require knowledge of the received amplitudes or the noise level. The simplest of such receivers is the single user MF receiver, which correlates the received signal with each of the signature signals from the set of signature signals.

A linear multiuser detector that exploits the multi-user interference without knowledge of the channel parameters is the decorrelator receiver. The decorrelator receiver correlates the received signal with each of the decorrelator signals $v_m(t)$ corresponding to the matrix

$$\mathbf{V} = \mathbf{S}(\mathbf{S}^*\mathbf{S})^{-1},$$

where **S** is the matrix corresponding to the signature signals $s_m(t)$. So $a_m = \langle v_m(t), r(t) \rangle$ an inner product which we wish to maximize for $1 \leq m \leq M$. For a mathematical discussion of the inner product, again see the Applicants' co-pending application "Correlation Shaping Matched Filter", U.S. Application No. 09/788,890, filed February 20, 2001.

It is known that a decorrelator receiver does not generally lead to optimal decisions, since in general the noise components in the outputs $a_m$ of the decorrelator receiver are correlated. This correlation is due to the fact that the outputs $a_m$ share information regarding the noise. Intuitively, it seems that eliminating this common (linear) information can improve the performance of the detector.

Let **a 40** denote the vector output of the decorrelator receiver. Then,

$$\mathbf{a} = \mathbf{V}^*\mathbf{r} = \mathbf{A}\mathbf{b} + \mathbf{V}^*\mathbf{n} \qquad \textbf{Equation 1}$$

where $\mathbf{A} = \mathrm{diag}(A_1,\ldots,A_M)$. The covariance of the noise component $\mathbf{V}^*\mathbf{n}$ in **a 40,** denoted $\mathbf{C}_a$, is

$$\mathbf{C}_a = \sigma^2\mathbf{V}^*\mathbf{V} = \sigma^2(\mathbf{S}^*\mathbf{S})^{-1}. \qquad \textbf{Equation 2}$$

Note that $\mathbf{C}_a$ is the covariance of $\mathbf{a} - \mathbf{a}'$ where $\mathbf{a}' = E(\mathbf{a}|\mathbf{b})$. Based upon the mathematics found in the Applicants' previously cited "Orthogonal Matched Filter Detection" reference, it follows that the noise components in **a** are uncorrelated if and only if the signature signals $S_m(t)$ are orthonormal. In this case, the decorrelator receiver does in fact lead to optimal decisions. To improve the detection performance when the signature signals are not orthonormal, without estimating the variance of the noise or the received amplitudes of the user's signals, one aspect of the invention whitens the output of the decorrelator receiver prior to detection, as depicted in **Figure 2**. It will be shown that this approach does in fact lead to improved performance over the MF detector and a conventional decorrelator receiver in many cases.

Suppose we whiten the vector output **a 40** of the decorrelator receiver using a whitening transformation (WT **50**) **W**, to obtain the random output vector $\mathbf{x} = \mathbf{W}\mathbf{a}$, where

the covariance matrix of the noise component in **x 60** is given by $\mathbf{C}_x = \sigma^2\mathbf{I}$, and then base our detection on **x 60**. We choose a WT **W** that minimizes the MSE given by

$$E_{mse} = \sum_{m=1}^{M} E((x'_m - a'_m)^2), \qquad \textbf{Equation 3}$$

where $a'_m = a_m - E(a_m|\mathbf{b})$ and $x'_m = x_m - E(x_m|\mathbf{b})$.

*EQUIVALENT PROBLEMS*

In this section, **Equation 3** is formulated in two equivalent ways that provide further insight into the problem. Specifically, we demonstrate that the following problems are the same:

Problem 1: Find an optimal WT **50 W** that minimizes the MSE defined above between the whitened output vector **x** = **Wa** and the input vector **a**, where **a 40** is the vector output of the decorrelator receiver.

Problem 2: Find a set of orthonormal signals $\{h_m(t), 1 \le m \le M\}$ that are closest in a least squares sense to the signals $\{v_m(t), 1 \le m \le M\}$, namely that minimize $\sum_m < (v_m(t) - h_m(t)) , (v_m(t) - h_m(t)) >$. Then correlate the received signal with each of the signals $h_m(t)$ to obtain the whitened output vector **x 60**.

Problem 3: Find a set of orthonormal signals that are closest in a least squares sense to the signature signals $\{s_m(t), 1 \le m \le M\}$. Then correlate the received signal with these signals to obtain whitened vector output **x 60**.

The remainder of this section discusses the equivalence between the problems above and their solution.

We first show that the detector depicted in **Figure 1** is equivalent to the detector of **Figure 2**, where the signals $h_m(t)$ are orthonormal and given by $h_m(t) = \sum_k \mathbf{W}^*_{km} v_k(t)$, where $\mathbf{W}^*_{km}$ denotes the $km^{\text{th}}$ element of $\mathbf{W}^*$.

The vector output **x 60** of the WT in **Figure 1** is given by

$$\mathbf{x} = \mathbf{Wa} = \mathbf{WV}^*\mathbf{r} = \mathbf{H}^*\mathbf{r}, \qquad \textbf{Equation 4}$$

where $\mathbf{H} = \mathbf{VW}^*$. Therefore, **x 60** can be viewed as the output of a bank of correlators **30** with signals $h_m(t) = \sum_k \mathbf{W}^*_{km} v_k(t)$, as depicted in **Figure 2**. Furthermore, employing **Equation 2** leads to $\mathbf{H}^*\mathbf{H} = \mathbf{W}\mathbf{V}^*\mathbf{V}\mathbf{W}^* = 1/\sigma^2 \mathbf{W}\mathbf{C}_a\mathbf{W}^* = 1/\sigma^2 \mathbf{C}_x = \mathbf{I}$, so that the signals $h_m(t)$ are orthonormal.

We will now demonstrate that the minimization of $E_{mse}$ given by **Equation 3** is equivalent to the minimization of the LSE $E_{ls}(v_m(t),h_m(t))$, where

$$E_{ls}(v_m(t),h_m(t)) = \sum_m < (v_m(t) - h_m(t)) \ ,(v_m(t) - h_m(t)) > . \qquad \textbf{Equation 5}$$

Using **Eqs. 4** and **1** results in

$\mathbf{x} - \mathbf{a} = (\mathbf{H} - \mathbf{V})^*\mathbf{r} = (\mathbf{H} - \mathbf{V})^*(\mathbf{Sab} + \mathbf{n})$,

and

$$x'_m - a'_m = <(h_m(t) - v_m(t)), n(t)>. \qquad \textbf{Equation 6}$$

Substituting **Equation 6** into **Equation 3** yields

$$E_{mse} = \sigma^2 \sum_{m=1}^{M} < (h_m(t) - v_m(t)),(h_m(t) - v_m(t)) > . \qquad \textbf{Equation 7}$$

Comparing **Equation 7** and **Equation 5** leads to the conclusion that the optimal whitening problem is equivalent to the problem of finding a set of orthonormal signals $h_m(t)$ that are closest in a least squares sense to the signals $v_m(t)$, establishing the equivalence of Problems 1 and 2.

Finally, Problems 2 and 3 may be shown to be equivalent by proving that the orthonormal signals $h_m(t)$ that minimize $E_{ls}(v_m(t),h_m(t))$ and $E_{ls}(s_m(t),h_m(t))$ are equal. To this end, we rely on the following lemmas.

*Lemma 1*: Let $\{y_m(t), 1 \le m \le M\}$ be a set of orthogonal signals with $<y_k(t),y_m(t)> = c^2_m \delta_{km}$, where $c_m > 0$ is arbitrary, and $\delta_{km} = 1$ when $k = m$ and 0 otherwise. Then the orthonormal signals $h_m(t)$ that minimize $E_{ls}(y_m(t),h_m(t))$ are $h_m(t) = y_m(t)/|c_m|$.

*Proof*: Since $<h_m(t),h_m(t)> = 1$, minimization of $E_{ls}(y_m(t),h_m(t))$ is equivalent to maximization of $\sum_{m=1}^{M} \langle h_m(t), y_m(t) \rangle$. Using the Cauchy-Swartz inequality,

$$\sum_{m=1}^{M}\langle h_m(t), y_m(t)\rangle \le \sum_{m=1}^{M}\left|\langle h_m(t), y_m(t)\rangle\right| \le \sum_{m=1}^{M}\langle y_m(t), y_m(t)\rangle^{1/2},$$

with equality if and only if $h_m(t) = y_m(t)/|c_m|$.

The following corollary results from *Lemma 1*.

*Corollary 1*: Let $\{y'_m(t) = d_m y_m(t), 1 \le m \le M\}$, where $d_m > 0$ are arbitrary constants and the signals $y_m(t)$ are orthogonal. Then the orthonormal signals $h_m(t)$ that minimize $E_{ls}(y_m(t),h_m(t))$ and $E_{ls}(y'_m(t),h_m(t))$ are the same.

*Lemma 2*: Let $y_m(t)$ and $y'_m(t)$ denote the columns of $\mathbf{Y}$ and $\mathbf{Y}' = \mathbf{YU}$ respectively, where $\mathbf{U}$ is an arbitrary unitary matrix. Let the columns of $\mathbf{H}$ and $\mathbf{H}'$ be the orthonormal signals $h_m(t)$ and $h'_m(t)$ that minimize $E_{ls}(y_m(t),h_m(t))$ and $E_{ls}(y'_m(t),h_m(t))$ respectively. Then $\mathbf{H}' = \mathbf{HU}$.

*Proof*: Since $(\mathbf{H}')^*\mathbf{H}' = \mathbf{U}^*\mathbf{H}^*\mathbf{HU} = \mathbf{I}$, the signals $h'_m(t)$ are orthonormal. The lemma then follows from

$$E_{ls}(y_m(t),h_m(t)) = \mathrm{Tr}((\mathbf{Y} - \mathbf{H})^*(\mathbf{Y} - \mathbf{H}))$$
$$= \mathrm{Tr}(\mathbf{U}(\mathbf{Y} - \mathbf{H})^*(\mathbf{Y} - \mathbf{H})\mathbf{U}^*) = E_{ls}(y'_m(t),h_m(t)).$$

Combining *Corollary 1* and *Lemma 2* it follows that if we find a unitary matrix such that the columns of $\mathbf{Y}' = \mathbf{VU}$ and $\mathbf{S}' = \mathbf{SU}$ are both orthogonal and proportional to each other, then the orthonormal signals minimizing $E_{ls}(v_m(t),h_m(t))$ and $E_{ls}(s_m(t),h_m(t))$ are the same. Let $\mathbf{S} = \mathbf{Q\Sigma Z}^*$ be the Singular Value Decomposition of $\mathbf{S}$, where $\mathbf{Q}$ and $\mathbf{Z}$ are unitary matrices and $\mathbf{\Sigma}$ is a diagonal $N \times M$ matrix with diagonal elements $\sigma_m > 0$. Then $\mathbf{V} = \mathbf{S}(\mathbf{S}^*\mathbf{S})^{-1} = \mathbf{Q}\tilde{\mathbf{\Sigma}}\mathbf{Z}^*$, where $\tilde{\mathbf{\Sigma}}$ is a diagonal $N \times M$ matrix with diagonal elements $1/\sigma_m$. Now, let $\mathbf{V}' = \mathbf{VZ}$ and $\mathbf{S}' = \mathbf{SZ}$. Then the columns $\mathbf{v}'_m(t)$ and $\mathbf{s}'_m(t)$ of $\mathbf{V}'$ and $\mathbf{S}'$ respectively, are both orthogonal, and $\mathbf{v}'_m(t) = d_m\mathbf{s}'_m(t)$ where $d_m = 1/\sigma_m^2$. Thus, the orthonormal signals minimizing $E_{ls}(v_m(t),h_m(t))$ and $E_{ls}(s_m(t),h_m(t))$ are the same.

This completes the proof that the three Problems outlined above are equivalent. The optimal whitening problem has been solved in its most general form in the Applicants' "Orthogonal Matched Filter Detection" reference cited above, from which it follows that the WT minimizing **Equation 3** is

$$\mathbf{W} = \sigma\mathbf{C}_a^{-1/2} = (\mathbf{S}^*\mathbf{S})^{1/2}.$$

The orthonormal signals that minimize $E_{ls}(v_m(t),h_m(t))$ and $E_{ls}(s_m(t),h_m(t))$ are then the columns of

$$\mathbf{H} = \mathbf{VW}^* = \mathbf{V}(\mathbf{S}^*\mathbf{S})^{1/2} = \mathbf{S}(\mathbf{S}^*\mathbf{S})^{-1/2}.$$

*WHITENING AND SUBSPACE WHITENING*

In one instance, the MMSE between the vector output **a 40** of a bank of correlators **30** comprising a matched filter receiver and the vector output **x 60** of a correlation shaper **50** comprising a whitening transformation **W** is achieved by employing a whitening transformation given by

$$\mathbf{W} = (\mathbf{S}^*\mathbf{S})^{-1/2}.$$

In another instance, the MMSE between the vector output **a 40** of a bank of correlators **30** comprising a matched filter receiver and the vector output **x 60** of a correlation shaper **50** comprising a subspace whitening transformation **W** is achieved by employing a subspace whitening transformation given by

$$\mathbf{W} = ((\mathbf{S}^*\mathbf{S})^{1/2})^{\dagger}.$$

In a third instance, the MMSE between the vector output **a 40** of a bank of correlators **30** comprising a decorrelator receiver and the vector output **x 60** of a correlation shaper **50** comprising a whitening transformation **W** is achieved by employing a whitening transformation given by

$$\mathbf{W} = (\mathbf{S}^*\mathbf{S})^{1/2}.$$

In a fourth instance, the MMSE between the vector output **a 40** of a bank of correlators **30** comprising a decorrelator receiver and the vector output **x 60** of a correlation shaper **50** comprising a subspace whitening transformation **W** is achieved by employing a subspace whitening transformation given by

$$\mathbf{W} = (\mathbf{S}^*\mathbf{S})^{1/2}.$$

*COVARIANCE MATRIX OF THE CORRELATION SHAPER OUTPUT IS ARBITRARY*

The correlation shaper **50** may be chosen so that the covariance matrix $\mathbf{C}_x$ of the output vector is arbitrary within the mathematical constraints imposed upon any covariance matrix. In this case, correlation shaper **50** may be chosen so that $\mathbf{W}\mathbf{C}_a\mathbf{W} = \mathbf{C}_x$, where $\mathbf{C}_a$ is the covariance matrix of the vector output $a$ **40** of the bank of correlators **30**. In this aspect, if the bank of correlators **30** comprises a matched filter receiver, then $\mathbf{C}_a = \mathbf{S}^*\mathbf{S}$. Alternatively, if the bank of correlators **30** comprises a decorrelator receiver, then $\mathbf{C}_a = (\mathbf{S}^*\mathbf{S})^{\dagger}$.

*RESTRICTION OF COVARIANCE MATRIX OF CORRELATION SHAPER OUTPUT TO PERMUTATION PROPERTY*

Correlation shaper **50** may be chosen so that the covariance matrix of output vector **x 60** has the property that the second row and each subsequent row is a permutation of the first row. Correlation shaper **50** may also be chosen so that the covariance matrix of output vector **x 60** when represented in subspace has the above property. The latter correlation shaper **50** may be referred to as a subspace correlation shaper.

A correlation shaper **50** that minimizes the MSE between the input and the output is given as follows. Let $\{d_k, 1 \leq k \leq M\}$ be the elements of the first row of the specified covariance matrix. Let **D** be a diagonal matrix whose diagonal elements are the square-roots of the generalized Fourier transform of the sequence $d_k$. The generalized Fourier transform is defined on a group formed by the elements of the prespecified covariance matrix. *See* Y.C. Eldar, G.D. Forney, Jr., "On Quantum Detection and the Square-Root Measurement", *IEEE Trans. on Inform. Theory*, vol. 47, No. 3, March 2001 (hereby incorporated by reference). Let **F** be a Fourier matrix representing the generalized Fourier transform over the group formed by the elements of the covariance matrix.

In a first embodiment, the MMSE between the vector output **a 40** of a bank of correlators **30** comprising a matched filter receiver and the vector output **x 60** of a correlation shaper **50** is achieved by employing a transformation given by

$$\mathbf{W} = \mathbf{SFD}(\mathbf{DF}^*\mathbf{S}^*\mathbf{SFD})^{-\frac{1}{2}}\mathbf{DF}^*.$$

In a second embodiment, the MMSE between the vector output **a 40** of a bank of correlators **30** comprising a matched filter receiver and the vector output **x 60** of a subspace correlation shaper **50** is achieved by employing a subspace transformation given by

$$\mathbf{W} = \mathbf{SFD}((\mathbf{DF}^*\mathbf{S}^*\mathbf{SFD})^{\frac{1}{2}})^{\dagger}\mathbf{DF}^*.$$

In a third embodiment, the MMSE between the vector output **a 40** of a bank of correlators **30** comprising a decorrelator receiver and the vector output **x 60** of a correlation shaper **50** is achieved by employing a transformation given by

$$\mathbf{W} = \mathbf{VFD}(\mathbf{DF}^*\mathbf{V}^*\mathbf{VFD})^{-\frac{1}{2}}\mathbf{DF}^*.$$

In a fourth embodiment, the MMSE between the vector output **a 40** of a bank of correlators **30** comprising a decorrelator receiver and the vector output **x 60** of a subspace correlation shaper **50** is achieved by employing a subspace whitening transformation given by

$$\mathbf{W} = \mathbf{VFD}((\mathbf{DF}^*\mathbf{V}^*\mathbf{VFD})^{\frac{1}{2}})^{\dagger}\mathbf{DF}^*.$$

*ORTHOGONAL & PROJECTED ORTHOGONAL, GEOMETRICALLY UNIFORM & PROJECTED GEOMETRICALLY UNIFORM CORRELATING SIGNALS*

In all cases, the closest signals in a least-squares sense to the signature signals are given by

$$q_k(\mathrm{t}) = \sum_{m=1}^{M} s_m(\mathrm{t})\mathbf{W}^*_{mk}$$

where **W** is the corresponding MMSE correlation shaper transformation, and $\mathbf{W}^*_{mk}$ is the $mk^{th}$ element of **W**. If the shaping signals are to be orthogonal signals, then a whitening transformation is to be used. If the shaping signals are to be projected

orthogonal signals, then a subspace whitening transformation is to be employed. If the shaping signals are geometrically uniform signals, a transformation that results in a covariance matrix with the permutation property is to be used. And for projected geometrically uniform shaping signals, a subspace correlation shaper with the permutation property should be used.

Similarly, the closest signals in a least-squares sense to the decorrelator signals are given by

$$q_k(t) = \sum_{m=1}^{M} v_m(t)\mathbf{W}^*_{mk}$$

where **W** is the corresponding MMSE correlation shaper transformation.

*II. SPECIFIC EMBODIMENTS*

*A. Orthogonal and Projected Orthogonal, Geometrically Uniform and Projected Geometrically Uniform Signals*

The following embodiments vary depending on the desired correlation shape and upon whether the received signals are linearly independent or linearly dependent. In each of the embodiments of this section, it may be desirable to minimize the MSE between vector output **x 60** of correlation shaper **50** and vector output **a 40** of the bank of correlators **30**.

*Linearly Independent Received Signals and a Decorrelated Output Vector*

In the first of these embodiments, it is assumed that the correlation shape chosen is to have the output vector **x 60** completely decorrelated, while the received signature signals are linearly independent. In this embodiment, the correlation shaper **50** performs a whitening transformation on the output vector **a 40**. After the whitening transformation **W**, the vector output **x 60** of correlation shaper **50**, which was correlated when it emerged from the bank of correlators **30**, becomes uncorrelated. This embodiment may perform

satisfactorily for a given system even if the correlation shaper does not result in the smallest MSE value between vector outputs **x 60** and **a 40**.

*Linearly Independent Received Signals and a Specified or Arbitrary Output Vector Correlation Shape*

In another embodiment, the vector output **x 60** of correlation shaper **50** may have a specified correlation shape. The correlation shape of vector output **x 60** may be altered by selecting the covariance matrix to have specific properties. In addition, one skilled in the art may decide in certain circumstances to allow the correlation shape of output vector **x 60** to be arbitrary. In this instance, the covariance matrix may be comprised of arbitrary values that satisfy the constraints imposed on any covariance matrix.

The specified covariance matrix of the vector output **x 60** may be selected to have the permutation property described above in which the second and each subsequent row is a permutation of the first.

This embodiment may perform satisfactorily for a given system even if selected correlation shaper **50** does not result in the smallest MSE value between vector outputs **x 60** and **a 40**.

*Linearly Dependent Received Signals and Decorrelated Output*

In another embodiment, vector output **x 60** may be decorrelated when the received signature signals are linearly dependent. When the received signals are linearly dependent, the components of vector output **a 40** of the bank of correlators **30** are deterministically linearly dependent, and consequently the components of **x** = **Wa** are also linearly dependent and cannot be statistically uncorrelated. Therefore, the linear dependence of the signature signals renders conventional whitening techniques impossible. Thus, in this alternative embodiment, vector output **a 40** of the bank of correlators **30** will be whitened on the subspace in which it lies. Subspace whitening may be defined such that the whitened vector lies in the subspace as specified in the

previously cited reference "Orthogonal and Projected Orthogonal MF Detection", and its representation in terms of any orthonormal basis for this is white subspace.